Polaritonic pulse and coherent *X*- and gamma rays from Compton (Thomson) backscattering

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The formation of polariton wave-packets created by high-intensity laser beams focused in plasmas is analyzed, and the velocity, energy, size, structure, stability, and electron content of such polaritonic pulses are characterized. It is shown that polaritonic pulses may transport trapped electrons with appreciable energies, provided the medium behaves as a rarefied classical plasma. The relativistic electron energy is related to the polariton group velocity, which is close to the velocity of light in this case. The plasma pulse is polarized, and the electron number in the pulse is estimated as being proportional to the square root of the laser intensity and the 3/2-power of the pulse size. It is shown that Compton (Thomson) backscattering by such polaritonic pulses of electrons may produce coherent *X*- and gamma rays, as a consequence of the quasirigidity of the electrons inside the polaritonic pulses and their relatively large number. The classical results of the Compton scattering are re-examined in this context, the energy of the scattered photons and their cross-section are analyzed, especially for backscattering, the great enhancement of the scattered flux of *X*- or gamma rays due to the coherence effect is highlighted and numerical estimates are given for some typical situations. © *2011 American Institute of Physics*. [doi:10.1063/1.3530599]

I. INTRODUCTION

It is well known that high-intensity laser pulses focused in a rarefied plasma can accelerate electrons up to considerable relativistic energies in the range of megaelectron volts or even gigaelectron volts.[1–13] Various models, both analytical and numerical, in particular the particle-in-cell simulations of such electron "bubbles," point toward the basic role played by plasmons and polaritons in laser-driven electron acceleration,[14–18] as it was suggested long ago.[19] It is widely agreed that the propagation of the laser radiation in plasma is governed by polaritonic excitations, arising from electrons interacting with the electromagnetic radiation. We give here a description of the formation of polaritonic wave-packets generated by high-intensity laser beams focused in a plasma, and characterize the velocity, energy, size, structure, stability, and electron content of such plasma pulses. It is shown that a coherent Compton (Thomson) backscattering by such high-energy pulses may result in brilliant fluxes of *X*- or gamma rays.

II. POLARITONIC PULSES

We consider the well-known plasma model consisting of electrons with density n, mass m, and charge $-e$ moving in a neutralizing, rigid (or quasirigid) background of positive ions. Let $\mathbf{u}(\mathbf{r},t)$ be a displacement field in electron positions, such as to create a small volume density imbalance $\delta n = -n \operatorname{div} \mathbf{u}$. We have, therefore, a charge density $\rho = en \operatorname{div} \mathbf{u}$ and a current density $\mathbf{j} = -en\dot{\mathbf{u}}$. The polarization electric ($\mathbf{E}$) and magnetic ($\mathbf{H}$) fields obey the Maxwell equations

$$\operatorname{div} \mathbf{E} = 4\pi en \operatorname{div} \mathbf{u}, \quad \operatorname{div} \mathbf{H} = 0,$$

$$\operatorname{curl} \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t}, \quad \operatorname{curl} \mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} - \frac{4\pi en}{c}\frac{\partial \mathbf{u}}{\partial t}, \tag{1}$$

where we assume a nonmagnetic plasma (i.e., the magnetization is zero, and the magnetic field is equal to the magnetic induction). It is easy to see that Eqs. (1) lead to

$$\frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} - \Delta \mathbf{E} = -4\pi en \operatorname{grad} \cdot \operatorname{div} \mathbf{u} + \frac{4\pi en}{c^2}\frac{\partial^2 \mathbf{u}}{\partial t^2}. \tag{2}$$

We assume that the effect of the pulsed electromagnetic fields on the electron motion is nonrelativistic, as a consequence of the high polarization field which may compensate to a large extent the original laser field. We assume therefore the Newton's law for the electron motion

$$m\ddot{\mathbf{u}} = -e\mathbf{E} - e\mathbf{E}_0 \tag{3}$$

under the action of the electric field, where $\mathbf{E}_0$ is the external electric field of the laser pulse. We note in Eq. (3) the absence of the Lorentz force and the approximation of the total time derivative with the partial time derivative, as for nonrelativistic motion.

Making use of Fourier transforms of the type

$$\mathbf{u}(\mathbf{r},t) = \frac{1}{(2\pi)^4}\int d\mathbf{k} d\omega \mathbf{u}(\mathbf{k},\omega) e^{i(\mathbf{kr}-i\omega t)}, \tag{4}$$

we get easily from Eqs. (2) and (3)

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$$\omega^2(\omega^2-\omega_p^2-c^2k^2)\mathbf{u}+\omega_p^2c^2\mathbf{k}(\mathbf{ku})=\frac{e}{m}(\omega^2-c^2k^2)\mathbf{E}_0, \quad (5)$$

where $\omega_p=\sqrt{4\pi ne^2/m}$ is the plasma frequency. Hence, we get

$$\omega^2(\omega^2-\omega_p^2-c^2k^2)\mathbf{u}=-\frac{e\omega_p^2c^2}{m}\mathbf{k}\frac{\mathbf{kE}_0}{\omega^2-\omega_p^2}+\frac{e}{m}(\omega^2-c^2k^2)\mathbf{E}_0, \quad (6)$$

where we can read the two well-known branches of elementary excitations: longitudinal plasmons, with frequency ω_p, and transverse polaritons, propagating with frequency $\omega_1=\sqrt{\omega_p^2+c^2k^2}$.[19] The plasmons do not "propagate," in the sense that their group velocity is vanishing, so we leave them aside. Or, more realistically, we assume that the external field is transverse ($\mathbf{kE}_0=0$), and get rid of the plasmon term in Eq. (6). We get therefore $\mathbf{ku}=0$ (and $\mathbf{kE}=0$), i.e., a vanishing volume charge density, as expected, and a transverse displacement field given by

$$\mathbf{u}=\frac{e}{m}\frac{\omega^2-c^2k^2}{\omega^2(\omega^2-\omega_1^2)}\mathbf{E}_0. \quad (7)$$

Making use of Eq. (3), we can see easily that the total field is given by $\mathbf{E}_{tot}=(m\omega^2/e)\mathbf{u}$, so we may define a "dielectric function" ε in this context, through $\mathbf{E}_0=\varepsilon\mathbf{E}_{tot}$, given by

$$\varepsilon(\mathbf{k},\omega)=\frac{\omega^2-\omega_1^2}{\omega^2-c^2k^2}. \quad (8)$$

It is convenient to introduce the vector potential $\mathbf{A}_0=-(ic/\omega)\mathbf{E}_0$ in Eq. (7), where we perform first the inverse Fourier transform with respect to the frequency, and retain only the ω_1-contribution. The full inverse Fourier transform of Eq. (7) reads

$$\mathbf{u}(\mathbf{r},t)=-\frac{e\omega_p^2}{4mc}\frac{1}{(2\pi)^3}\int d\mathbf{k}\frac{1}{\omega_1^2}\mathbf{A}_0(\mathbf{k},\omega_1)e^{i(\mathbf{kr}-\omega_1 t)}. \quad (9)$$

We focus now, in Eq. (9), on a certain wave vector $\mathbf{k}_0$ and perform a series expansion of ω_1 in powers of $\mathbf{q}=\mathbf{k}-\mathbf{k}_0$, where $0<q<q_c$, the cutoff wave vector q_c being such that $q_c\ll k_0$. We assume an isotropic cutoff wave vector. As it is well known, we get an isotropic wave-packet extending approximately over the length $d=2\pi/q_c\gg\lambda_{10}$, where λ_{10} is the wavelength of the wave with frequency $\omega_{10}=\sqrt{\omega_p^2+c^2k_0^2}$. This pulse is propagating with the group velocity $\mathbf{v}=\partial\omega_1/\partial\mathbf{k}$ for $\mathbf{k}=\mathbf{k}_0$. We get easily

$$\mathbf{v}=\frac{c^2\mathbf{k}_0}{\sqrt{\omega_p^2+c^2\mathbf{k}_0^2}} \quad (10)$$

and the displacement field $\mathbf{u}(\mathbf{r},t)$ from Eq. (9) which can be represented as

$$\mathbf{u}(\mathbf{r},t)\simeq-\frac{e\omega_p^2}{4mc\omega_{10}^2}\mathbf{A}_0(\mathbf{k}_0,\omega_{10})\delta(\mathbf{r}-\mathbf{v}t). \quad (11)$$

It is worth emphasizing that the δ-function of the pulse is in fact a representation for a function of the type $\sim(\sin q_cx/x)\times(\sin q_cy/y)(\sin q_cz/z)$, along the propagation direction, where $x=r-vt$ is the coordinate along the pulse motion and y, z denote the transverse coordinates, perpendicular to the direction of motion. We can see that this function is localized over the volume $\sim d^3$, and has a peaked height $\sim q_c^3\sim 1/d^3$.

Let us assume $\omega_p\ll\omega_0=ck_0$. Then, Eq. (10) gives a group velocity

$$v\simeq c\left(1-\frac{\omega_p^2}{2\omega_0^2}\right), \quad (12)$$

which is close to the velocity of light c. An electron trapped in such a pulse gets an energy

$$E_{el}=\frac{mc^2}{\sqrt{1-v^2/c^2}}\simeq\frac{\omega_0}{\omega_p}mc^2. \quad (13)$$

For realistic values $\hbar\omega_0=1$ eV ($\lambda_0=2\pi c/\omega_0\simeq 1$ μm and $\hbar$ is Planck's constant) and an electron density $n=10^{18}$ cm^{-3} we get $\hbar\omega_p=3\times10^{-2}$ eV and $E_{el}\simeq 17$ MeV.

With this assumption the displacement in the pulse given by Eq. (11) can be written as

$$\mathbf{u}_0\simeq-\frac{e\omega_p^2}{4mc\omega_0^2d^3}\mathbf{A}_0(\mathbf{k}_0,\omega_0). \quad (14)$$

It is easy to see that a similar pulse is obtained for the vector potential $\mathbf{A}_0$. We take it of the form

$$\mathbf{A}_0(\mathbf{r},t)=\mathbf{A}_0d^3\delta(\mathbf{r}-\mathbf{v}t), \quad (15)$$

where $\mathbf{A}_0$ is real. It consists of a superposition of frequencies in the range $\Delta\omega=cq_c=2\pi c/d$, so we have approximately $\mathbf{A}_0(\mathbf{k}_0,\omega_0)\simeq\mathbf{A}_0d^4/c$ and get finally

$$\mathbf{u}_0\simeq-\frac{e\omega_p^2d}{4mc^2\omega_0^2}\mathbf{A}_0. \quad (16)$$

As we said above, the displacement $\mathbf{u}_0$ is transverse ($\mathbf{k}_0\mathbf{u}_0=0$), and there is no volume charge density in the pulse. The charge is distributed transversally toward the pulse surface. Let us assume that this distribution extends over a region of thickness l; then, we may take approximately $\delta n_0=nu_0/l$ for the electron density imbalance, where l is of the order of the wavelength λ_0, for a perfect δ-pulse. We get the total number of electrons in the pulse

$$N\simeq\pi nd^3\frac{e\omega_p^2}{4mc^2\omega_0^2}A_0, \quad (17)$$

where we can see that N does not depend on the thickness l. It is convenient to express the vector potential A_0 by means of the density of the field energy $w_0=k_0^2A_0^2/4\pi$. In addition, we introduce the notations $\varepsilon_p=\hbar\omega_p$, $\varepsilon_0=\hbar\omega_0$, and $\varepsilon_{el}=e^2/d$, the later being the Coulomb energy of an electron localized in the pulse. We get

$$N=nd^2\lambda_0\frac{\varepsilon_p^2}{4mc^2\varepsilon_0^2}\sqrt{\pi\varepsilon_{el}W_0}, \quad (18)$$

where W_0 is the total amount of field energy in the pulse ($W_0=I_0d^3/c$, where I_0 is the laser intensity).

For typical values $I_0=10^{18}$ W/cm^2, $d=1$ mm ($W_0=10^{23}$ eV and $\varepsilon_{el}=10^{-6}$ eV), $n=10^{18}$ cm^{-3} ($\varepsilon_p=3$

$\times 10^{-2}$ eV), $\varepsilon_0=1$ eV ($\lambda_0 \simeq 1$ μm), and $mc^2=0.5$ MeV we get $N \simeq 10^{11}$ electrons in the pulse, transported with the energy $\simeq 17$ MeV. Their total energy is $W_{el} \simeq 10^{18}$ eV, the remaining energy (up to $W_0=10^{23}$ eV) being left in the polarized laser pulse. Numerical data from recent experimental measurements[11–13] seem to be in fair agreement with Eqs. (13) and (18) given here.

It is worth noting the $\sqrt{W_0}$ ($\sim \sqrt{I_0}$)-dependence of the total number of electrons in the pulse. Making use of Eqs. (13) and (18), we can estimate the electron energy per pulse $W_{el}=E_{el}N$ and the efficiency coefficient

$$\eta = \frac{W_{el}}{W_0} = nd^2\lambda_0 \frac{\varepsilon_p}{4\varepsilon_0}\sqrt{\frac{\pi\varepsilon_{el}}{W_0}}, \tag{19}$$

which goes like the inverse square root of the field energy (laser intensity). Equation (19) sets a limit on the present approach, given by $\eta=1$. In this limit (which can be approached, for instance, by decreasing the pulse energy W_0), the electrons in the pulse are as rarefied [less than one electron per wavelength λ_0, according to Eqs. (18) and (19)] as the polarization inside the pulse becomes ineffective, and the pulse cannot be treated anymore as a macroscopic piece of matter. It is also worth noting the $d^{3/2}$-dependence of the number of electrons in the pulse, which may induce the temptation of increasing the pulse size in order to increase the efficiency. However, for high values of the size d, the energy is distributed in fact over many such pulses, with smaller heights; it is spread over large portions of the sample in fact, and we get finally a net decrease in efficiency.

Using the same numerical values as above we can estimate the displacement given by Eq. (16) as $u_0=N/\pi nd^2 \simeq 10^{-2}$ μm, which is a very small displacement, as expected. The pulsed fields acquire a very small frequency, arising from the factor $e^{i(\omega_{10}t-\mathbf{k}_0\mathbf{r})}$ which is omitted in Eqs. (11) and (14). Since $\omega_{10}=\sqrt{\omega_p^2+\omega_0^2}$, it is easy to see that this frequency is of the order of $\Omega=\omega_p^2/2\omega_0$, so the electron velocity in the pulse is of the order of $\Omega u_0=\omega_p^2 u_0/2\omega_0$. This is a very small velocity in comparison with the velocity of light ($\simeq 10^{-4}c$), which justifies the nonrelativistic approximation in treating the electron motion. The polarization charge oscillates slowly in the pulse, with a small phase velocity, $v_f=\Omega/k_0=c\omega_p^2/\omega_0^2 \simeq 10^{-3}c$. It is the trapped motion carried along by the pulse that made the electrons to acquire relativistic velocities. This motion is decoupled from the displacement $\mathbf{u}$, it pertains to the pulse coordinate $\mathbf{r}$. The motion of the electrons as described here is an inertial transport. The charge is polarized by the external field E_0 and the electrons are kept inside the pulse by the polarization field E.

The effect of the polarization can be seen in another way, by estimating the motion of the electrons inside the pulse under the action of the external field. In this case, we must use the relativistic equation of motion. We have therefore $m\ddot{u}/(1-v^2/c^2)^{1/2}=-eE_0$, and an amplitude $u_0^0=eE_0(1-v^2/c^2)^{1/2}/m\omega_0^2$ of the displacement produced by the bare, external field E_0. For the numerical values given above, the external electric field is $E_0 \simeq 10^{12}$ V/m (10^7 statvolt/cm) [and the external magnetic field is $H_0 \simeq 10^3$ Ts (10^7 G s)]. We get $u_0^0 \simeq 10^{-3}$ μm$=0.1u_0$, where u_0 is the displacement computed above for the total field $E_{tot}=E_0+E$. Making use of the above equation of motion, we can estimate the total field as $E_{tot}=(\Omega/\omega_0)^2 u_0 E_0/u_0^0 \simeq 10^{-6}E_0$, which shows that the polarization field practically cancels out the external field. The total field inside the pulse is almost vanishing, i.e., the polarization is highly effective in the pulse, which justifies again the use of the nonrelativistic equation of motion for the internal motion of the electrons inside the pulse. The quantity $(\omega_0/\Omega)^2 u_0^0/u_0 \simeq 10^6$, which is very large, can be viewed as the dielectric constant of the plasma in the pulse. Indeed, turning back to the dielectric function given by Eq. (8), we can see that for $k \simeq k_0$ and $\omega \simeq \omega_0=ck_0$, this dielectric function becomes infinitely large, which implies that the total field is vanishing. The electrons move practically in-phase with the polaritonic pulse.

The dielectric function discussed above is the "microscopic" dielectric function, pertaining to microscopic fields which affect the electron motion. We can define a macroscopic dielectric constant, by using Eq. (8), and following the steps leading to the pulse. We get easily $\varepsilon \simeq \omega_p^2 d/c\omega_0$, which, for our numerical values given above, is of the order of unity. This latter dielectric function is effective in the motion of an external (unpolarized) electron affected by the pulse, which experiences a high field, of the order of the external field E_0.

Even in such an ideal situation as the one discussed here, the pulse may still have a dispersion. As it is well known, one source of dispersion originates in high-order contributions in the q-expansion of the frequency around the wave vector k_0. This dispersion flatens gradually the pulse. Another source of a sui-generis dispersion may arise from fluctuations in the plasma density, which are of the order of n (or simply from plasma inhomogeneities). These fluctuations induce a corresponding dispersion in the plasma frequency and the group velocity of the pulse, so that we may speak in fact of a set of pulses, propagating with various velocities. It is easy to see that such an effect may give rise to a dispersion in the electron energy of the order of E_{el}.

Finally, it is worth commenting on another point. The positive ions are rigid (or quasirigid) in comparison with the electrons which are mobile. While the latter are carried along by the pulse, the former will be depolarized by a wakefield and an electron backflow, which give rise to plasma oscillations outside the pulse. This is the well-known picture of wakefield accelerated electrons, and the related bubble models.[1,14–18] Therefore, the pulse energy is also spend for creating these depolarizing plasma oscillations in the sample, as expected. An unpolarized electron in the process of being accelerated by the pulse will experience an uncompensated field of the order of E_0 (or the compensating polarization field E). The energy gain E_{el} of an accelerated electron is therefore obtained by the work of the force eE_0 over a distance δ. With our numerical values used here we get $\delta \simeq 10$ μm, which may give an estimate for the surface thickness l of the pulse (or the contrast thickness of the pulse).

III. COMPTON (THOMSON) BACKSCATTERING

The propagating polaritonic pulse is polarized, in the sense that the mobile electrons in the propagating pulse are

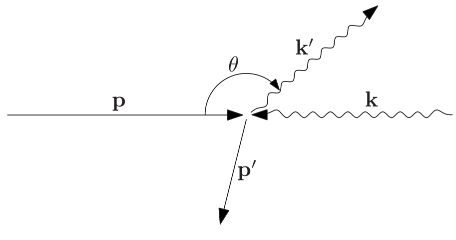

FIG. 1. Head-on electron-photon collision.

displaced from their equilibrium positions with respect to the quasirigid background of positive ions, such that the polarization field compensates, practically, the laser field. The electrons inside the pulse accumulate on the surface of the pulse, along a direction which is transverse to the direction of pulse propagation (laser radiation is transverse), such as a new equilibrium is reached, in the presence of the laser field. The polarized, equilibrium electrons in the polaritonic pulse are practically quasirigid (subjected to very slow density oscillations). They are carried along by the pulse in an inertial motion, while the quasirigid ions are depolarized by a wakefield and an electron backflow, which give rise to plasma oscillations outside the pulse. This is the well-known picture of wakefield accelerated electrons, and the related bubble models, as derived by various theoretical models and numerical simulations.[1,14–18]

The quasirigid electrons in the polaritonic pulse moving with relativistic velocities offer a unique opportunity of coherent Compton backscattering, which may produce coherent high-energetic *X*- or even gamma rays, i.e., an *X*-ray or gamma-ray laser.

The Compton scattering of gamma rays by a moving electron is shown schematically in Fig. 1. With usual notations $p=(E,\mathbf{p})$ and $k=(\omega,\mathbf{k})$ are the electron and, respectively, photon 4-momenta, and we set $c=\hbar=1$. $E=E_{el}$ denotes here the electron energy (not to be mistaken for the electric field). We assume a head-on (unpolarized) collision. From the momentum-energy conservation $p+k=p'+k'$, written as $p'=p+k-k'$, we get $pk-pk'-kk'=0$, or, making use of $p^2=p'^2=m^2$, $k^2=k'^2=0$

$$\omega'=\omega\frac{E+|\mathbf{p}|}{E+|\mathbf{p}|\cos\theta+\omega(1-\cos\theta)}. \tag{20}$$

Since $|\mathbf{p}|=vE=mv/\sqrt{1-v^2}$, this equation can also be written as

$$\omega'=\omega\frac{1+v}{1+v\cos\theta+\gamma\sqrt{1-v^2}(1-\cos\theta)}, \tag{21}$$

where $\gamma=\omega/m$ and v is the velocity of the electron (velocity of the polaritonic pulse). For all relevant situations (except ultrarelativistic limit), the inequality $2\gamma\sqrt{1+v}\ll\sqrt{1-v}$ is satisfied (Thomson scattering). The ratio ω'/ω given by Eq. (21) versus angle θ is shown in Fig. 2 in this case $[4\gamma^2(1+v)\ll 1-v]$ for a few values of the parameter v. The maximum value of the frequency ω' of the scattered photon is obtained for the scattering angle $\theta\simeq\pi$ (backscatering). This increase is sometimes assigned to a Doppler effect, which would introduce a relativistic factor $4/(1-v^2)\simeq(1+v)/(1-v)\simeq 2/(1-v)$ for $v\simeq 1$. For the typical parameter values used in this paper, $1-v\simeq\omega_p^2/2\omega_0^2\simeq 4.5\times 10^{-4}$, which is much greater than $2\gamma\sqrt{1-v^2}\simeq 10^{-7}$ (we take the frequency of the incident photon $\omega=1$ eV, $\gamma\simeq 2\times 10^{-6}$). Therefore, we may neglect the γ-term in Eq. (21), and get a maximum scattered frequency

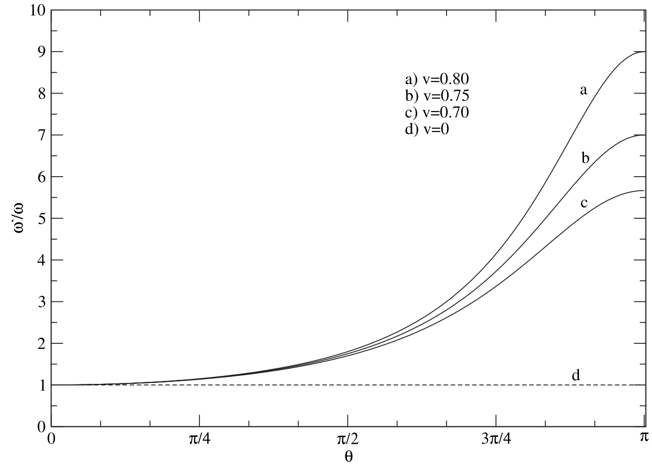

FIG. 2. The ratio of the energy of the scattered photon to the energy of the incident photon vs scattering angle for a few values of the polariton (electron) velocity v [Eq. (21) for $1-v\gg 4\gamma^2(1+v)$].

$$\omega'\simeq\omega\frac{1+v}{1-v}\simeq 10\ \text{keV}, \tag{22}$$

for the backscattering angle $\theta=\pi$. It is easy to see that an increase by an order of magnitude in the energy of the accelerated electrons $(E=E_{el}\simeq m\omega_0/\omega_p)$ means a decrease by two orders of magnitude in $1-v$ $(1-v\simeq\omega_p^2/2\omega_0^2)$, such that, by Eq. (22), we may get $\omega'\simeq 1$ MeV for the frequency of the backscattered gamma rays. Such high backscattering frequencies are concentrated around $\theta=\pi$ within a range $\Delta\theta\simeq\sqrt{2(1-v)/3v}$.

The well-known Compton cross-section can be written as[20]

$$d\sigma=8\pi r_e^2\frac{m^2d(-t)}{(s-m^2)^2}\left[\left(\frac{m^2}{s-m^2}+\frac{m^2}{u-m^2}\right)^2+\frac{m^2}{s-m^2}+\frac{m^2}{u-m^2}-\frac{1}{4}\left(\frac{s-m^2}{u-m^2}+\frac{u-m^2}{s-m^2}\right)\right], \tag{23}$$

where $r_e=e^2/m$ is the classical electron radius and

$$s=(p+k)^2=m^2+2pk,\quad u=(p-k')^2=m^2-2pk',$$

$$t=(k'-k)^2=-2kk' \tag{24}$$

are the invariant kinematical variables. By straightforward calculations this expression can be put in the form

$$d\sigma=\pi r_e^2\frac{(1-v^2)\sin\theta d\theta}{[1+v\cos\theta+\gamma\sqrt{1-v^2}(1-\cos\theta)]^2}\times\left[\left(\frac{v+\cos\theta}{1+v\cos\theta}\right)^2+\gamma\sqrt{1-v^2}\frac{1-\cos\theta}{1+v\cos\theta}+\frac{1+v\cos\theta}{1+v\cos\theta+\gamma\sqrt{1-v^2}(1-\cos\theta)}\right], \tag{25}$$

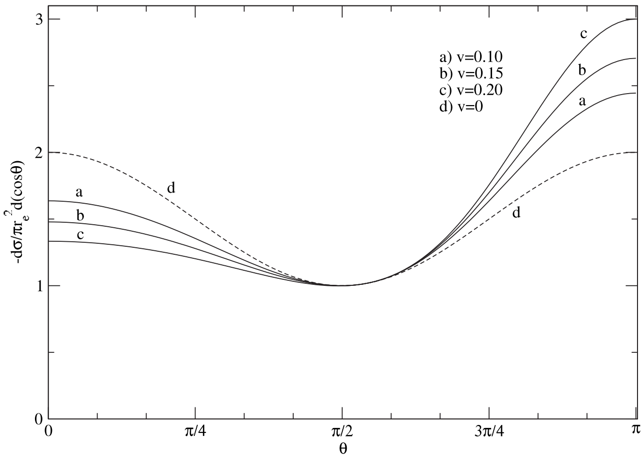

FIG. 3. Compton cross-section vs scattering angle for a few values of the polariton (electron) velocity v [Eq. (26), $1-v \gg 4\gamma^2(1+v)$].

where the transport velociy v is shown explicitly. Similarly, for the parameter values used here we may neglect the γ-terms in Eq. (25) (Thomson scattering), and get

$$d\sigma \simeq \pi r_e^2 \frac{1-v^2}{(1+v\cos\theta)^2}\left[\left(\frac{v+\cos\theta}{1+v\cos\theta}\right)^2+1\right]\sin\theta d\theta. \tag{26}$$

This cross-section is shown in Fig. 3 for a few values of the parameter v. The total backscattering cross-section is given by

$$\sigma_b \simeq \left. \pi r_e^2 \frac{1-v^2}{(1+v\cos\theta)^2}\left[\left(\frac{v+\cos\theta}{1+v\cos\theta}\right)^2+1\right]\right|_{\theta=\pi} \times\Delta(-\cos\theta) = \pi r_e^2 \frac{1+v}{1-v}(\Delta\theta)^2 \simeq 4\pi r_e^2/3 \tag{27}$$

and the rate of the backscattered photons is $dN_{ph}/dt = c\sigma_b n_{ph}$, where n_{ph} is the photon density in the incident flux. The energy loss of the scattered (recoil) electron for backscatering is $\Delta E=\omega'-\omega \simeq 2\omega v/(1-v)$ ($\Delta E/E \simeq 2\gamma v\sqrt{(1+v)/(1-v)} \ll 1$), which is approximately equal with the energy of the scattered photon $\omega' \simeq \omega(1+v)/(1-v)$ given above for $v\simeq 1$ (since $\omega \ll \omega'$). The momentum transferred to the electron in the scattering process is very small, in comparison with the initial momentum of the electron. For a polaritonic pulse, this momentum is transferred to the whole ensemble of electrons, as a consequence of the rigidity of the electrons in the polaritonic pulse. For the sake of the comparison, we note that the total cross-section is $8\pi r_e^2/3 \simeq 2\sigma_b$, as it is well known.

The cross-section computed above refers to one electron (and one photon). The field bispinors in the interaction matrix element (the scattering amplitude) between the initial state and the final state are normalized to unity. If we have N electrons, then each of them contributes individually to the cross-section, which is multiplied by N (i.e., $\sigma_b \to N\sigma_b$). This is an incoherent scattering. For the electrons in the polaritonic pulse, the situation is different. These electrons are not independent anymore (because of their rigidity inside the pulse), and they suffer the scattering collectively. This amounts to normalize the bispinors to N, such that each bispinor carries now a factor $\sqrt{N}$. Consequently, the scattering amplitude acquires an additional factor N and the cross-section acquires an additional factor N^2. In comparison with the incoherent scattering we get an additional factor N in the coherent scattering, which increases considerably the cross-section for large values of N.[21–25]

From the above estimations, we can see that the energy of the backscattered photons is much higher than the energy of the incident photons. Therefore, in the following estimations we can neglect the energy of the incident photons. The energy of the scattered photons is produced at the expense of the energy of the electrons. By successive Compton scattering we may expect a certain limitation on the duration of the scattering process for electron pulses (beside the limitations caused by the pulse duration, both for the electrons and the incident photons). Such a limitation is more stringent for the coherent scattering (due to the occurrence of the factor N^2).

Making use of the rate $d^2N_{ph}/d\theta dt = c(d\sigma/d\theta)n_{ph}$ of the scattered photons we can write down the rate of the energy produced by Compton (Thomson) scattering

$$dE^{coh} = \left(\int d\theta\omega' dN_{ph}/d\theta\right)dt = N^2 c n_{ph}\left(\int \omega' d\sigma\right)dt. \tag{28}$$

The integral in Eq. (28) can be computed by using ω' given by Eq. (21) (with $\gamma=0$) and the differential cross-section given by Eq. (26). The result is

$$dE^{coh} = \frac{8\pi}{3}N^2\omega c r_e^2 n_{ph}\frac{1}{1-v}dt. \tag{29}$$

This energy must be compared with the energy loss of the electrons in the polaritonic pulse

$$-NdE = -Nmd\frac{1}{\sqrt{1-v^2}}. \tag{30}$$

Integrating the equation $dE^{coh}=-NdE$ with the new variable $x=m/E$, we get easily

$$\frac{8\pi}{3}N\omega c n_{ph}\Delta t = m\int_{x_0}^1 dx \frac{1}{1+\sqrt{1-x^2}}, \tag{31}$$

where $x_0=m/E_0 \ll 1$ corresponds to the initial energy (E_0) of the polaritonic pulse. The integral in Eq. (31) can easily be estimated ($\simeq \pi/2-1$), so we get the duration Δt of the scattering

$$\Delta t \simeq (\pi/2-1)\frac{3mc}{8\pi N\hbar\omega r_e^2 n_{ph}}, \tag{32}$$

where we have re-established in full the universal constants.

We assume an incident flow of photons with intensity $I = 10^{14}$ W/cm^2 focused on a spatial region of size $d=1$ mm (picosecond pulses); the energy is $W=Id^3/c \simeq 3$ J and, for photon energy $\omega=1$ eV, we get a photon density $n_{ph} \simeq 5 \times 10^{22}$ cm^{-3}. For $N=10^{11}$ given before for the polaritonic pulse (and $r_e=2.8\times 10^{-13}$ cm) we get $\Delta t \simeq 10^{-15}$ s (femtoseconds). This time is an estimate for the duration of the collision, and for the duration of emission of the backscattered photons. As we can see, it does not depend, practically, on the electron energy in the polaritonic pulse (E_0), for high, relativistic energies. It is expected that the polaritonic pulse is "stopped," and, in fact, destroyed, after the lapse of this time.

The total energy of the backscattered photons can be estimated similarly, by using Eq. (28) and dt/dE from $dE^{coh}=-NdE$, where dE^{coh} is given by Eq. (29). Let us assume that we are interested in the photon backscattering within an angle $\Delta\theta=\alpha\sqrt{2(1-v)/3v}$, with $\alpha \ll 1$. Then, we get easily

$$E_b^{coh} = \frac{1}{4}N\alpha^2 \int_m^{E_0} \frac{(1+v)^2}{v} dE, \tag{33}$$

and, following the same technique as above, we get $E_b^{coh} \simeq \alpha^2 N E_0$, where we can recognize the total energy of the polaritonic pulse $W_{el}=NE_0$. This result is valid for $\alpha \ll 1$. For high, relativistic velocities $\alpha \simeq 1$, and practically the whole polaritonic energy is recovered in the backscattering photons.

IV. CONCLUDING REMARKS

In conclusion, we may say that polaritonic pulses of electrons transported by high-intensity laser radiation focused in a rarefied plasma may serve as targets for coherent Compton backscattering in the *X*-rays or gamma-rays energy range, therefore as a means for obtaining an *X*-ray or gamma-ray laser. The coherent scattering, which enhances considerably the photon output and ensures its coherence, is due to the quasirigidity of the electrons in the propagating polaritonic pulse, which ensures (within certain limits) the stability of this interacting formation of matter and electromagnetic radiation. The energy and cross-section of the Compton (Thomson) backscattering was re-examined in this paper in the context of the coherent scattering by polaritonic pulses, and the (pulse) duration of the backscattering emission was also estimated. Similar ideas have been advanced recently, especially for laser-driven accelerated electron mirrors.[26–32]

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