PLASMA FREQUENCY IN A LAYERED ELECTRON GAS

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The equation-of-motion method is applied in the RPA for an electron-hole pair operator in the layered electron gas of Visscher and Falicov. The band of plasma frequencies is obtained at zero temperature.

Some years ago Visscher and FAlicov have investigated the static-dielectric screening properties of a layered electron gas [1]. The system consists of a series of n parallel equally spaced planes, the distance between two neighbouring planes being a. A number of N electrons per unit area is allowed to move freely in each plane but there is no special term of tunneling between planes. A uniform rigid background of positive charge exists in each plane, whose density is equal to the average electron density.

The aim of this letter is to calculate the plasma frequency at zero temperature in this layered structure of electrons.

Recently, Fetter reported a result of this kind [2]. But he used a macroscopic model of layered electron fluid and an electro-hydrodynamic description. He found the band of frequencies

$$\omega^2(\kappa, k) = \omega_{\mathrm{p}}^2 \frac{ak}{2} \frac{\sinh ak}{\cosh ak - \cos a\kappa} + s^2k^2, \tag{1}$$

where κ and k stand, respectively, for the perpendicular to the plane and in-plane components of the wave vector, $\omega_{\mathrm{p}}^2 = 4\pi Ne^2/Ma$ is the usual bulk plasma frequency and s is the adiabatic speed of sound in the two-dimensional Fermi gas of electrons. A microscopic model and a many-body technique were used by Grecu in a previous paper [3]. The plasma frequency obtained by him is given by

$$\omega^2(\kappa, k) = \omega_{\mathrm{p}}^2\, k^2/(\kappa^2 + k^2), \tag{2}$$

which is the long-wavelength limit ($\kappa \to 0, k \to 0$) of eq. (1).

We shall obtain the first term of the plasma frequency (eq. (1)) by the equation-of-motion method. The single-particle wave functions of the unperturbed eigenstates of electrons are given by

$$\varphi_{m\boldsymbol{k}}(x, \boldsymbol{r}) = \chi(x - ma) \exp(\mathrm{i}\,\boldsymbol{k}\cdot\boldsymbol{r}), \tag{3}$$

where $\boldsymbol{r} = (y, z)$ and m labels the planes. The spin is disregarded for simplicity. The function $\chi(x - ma)$ is arbitrarily highly localized on the mth site and it is effectively the square root of a $\delta(x - ma)$ function [1]. The representation $\delta(x) = (na)^{-1} \times \Sigma_\kappa \exp(\mathrm{i}\kappa x)$ will be used for the $\delta(x)$ function, where $\kappa = (2\pi/na)p$, p any integer (cyclic boundary conditions). The functions (3) are normalized to unity $(\varphi_{m\boldsymbol{k}}, \varphi_{m'\boldsymbol{k}'}) = \delta_{mm'}\delta_{\boldsymbol{k}\boldsymbol{k}'}$. The Hamiltonian containing the kinetic and Coulomb interaction terms is

$$H = \sum_{m\boldsymbol{k}} \epsilon_k a^+_{m\boldsymbol{k}} a_{m\boldsymbol{k}} + M(2nN)^{-1} \sum_{\substack{m_1 m_2 \\ \kappa\boldsymbol{k}\boldsymbol{k}_1\boldsymbol{k}_2}} k^{-2}\omega^2(\kappa, k) \exp[\mathrm{i}\kappa(m_1 - m_2)a]\; a^+_{m_1\boldsymbol{k}_1+\boldsymbol{k}} a_{m_2\boldsymbol{k}_2-\boldsymbol{k}} a_{m_2\boldsymbol{k}_2} a_{m_1\boldsymbol{k}_1}, \tag{4}$$

where $\epsilon_k = \hbar^2k^2/2M$, $\omega^2(\kappa, k)$ is given by eq. (2) and $a_{m\boldsymbol{k}}(a^+_{m\boldsymbol{k}})$ is the annihilation (creation) operator of an electron localized on the plane m with the wave vector $\boldsymbol{k}$. The exclusion of $k = 0$ takes into account the uniform positive background. The electron density operator $\rho_{-(\kappa\boldsymbol{k})}$ is given by

$$\rho_{-}(\kappa \boldsymbol{k}) = \sum_{m\boldsymbol{k}_1} \exp(\mathrm{i}\kappa m a)\, a^{+}_{m\boldsymbol{k}_1+\boldsymbol{k}} a_{m\boldsymbol{k}_1}. \tag{5}$$

We shall require a time dependence of the form $\exp(\mathrm{i}\omega t)$ for the electron-hole pair operator $\Sigma_m \exp(\mathrm{i}\kappa m a)\, a^{+}_{m\boldsymbol{k}_1+\boldsymbol{k}} a_{m\boldsymbol{k}_1}$.

Using the equation-of-motion method in the RPA the calculations are standard and one obtains

$$k^2 D^{-1}(\omega, k)\, \rho_{-(\kappa\boldsymbol{k})} = \sum_{G} \omega^2(\kappa + G, \boldsymbol{k})\, \rho_{-(\kappa+G\boldsymbol{k})}, \tag{6}$$

where $G = (2\pi/a)p$, p any integer, are vectors in the reciprocal lattice of our layered structure and

$$D(\omega, k) = MN^{-1} \sum_{\boldsymbol{k}_1} (n_{\boldsymbol{k}_1} - n_{\boldsymbol{k}_1+\boldsymbol{k}})/(\hbar\omega + \epsilon_{k_1} - \epsilon_{k_1+k}), \tag{7}$$

$n_{\boldsymbol{k}} = \langle a^{+}_{m\boldsymbol{k}} a_{m\boldsymbol{k}} \rangle$ being the Fermi distribution at zero temperature for the two-dimensional electron gas in each plane For small values of k the leading term in eq. (7) is k^2/ω^2, so that the plasma frequency is obtained as

$$\omega^2 = \sum_{G} \omega^2(\kappa + G, k) = \omega_{\mathrm{p}}^2 \sum_{G} k^2/[(\kappa + G)^2 + k^2] \tag{8}$$

taking into account that $\rho_{-(\kappa+G\boldsymbol{k})} = \rho_{-(\kappa\boldsymbol{k})}$, according to eq. (5). The summation over G is easy to perform. We get [4]

$$\omega^2 = \omega_{\mathrm{p}}^2 \frac{ak}{2} \sinh ak/(\cosh ak - \cos a\kappa), \tag{9}$$

which is the first term in eq. (1). Addition k-dependent terms can be obtained by taking the next terms in the expansion of $D(\omega, k)$ [5]. On deriving the plasma frequency (9) we have not considered the interaction between electrons and plasmons [6].

References

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