CLUSTER RADIOACTIVITY
PAST, PRESENT and FUTURE

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MOTTO:

“The goal of science ... is to discover simplicity in the midst of complexity”

OUTLINE

- Historical milestones
- Macroscopic-microscopic method
- Unified approach of cold fission, $\alpha$-decay and heavy ion radioactivities within ASAF model
- Experimental confirmations
- Fine structure
- Extensions
  - saddle-point shapes obtained as solution of an Euler-Lagrange equation
  - $\alpha$-decay of superheavies (ASAF, universal curve, semi-empirical formula)
  - multicluster fission (true ternary, quaternary, etc)
  - atomic cluster on a surface (with A. Solov’yov and R. A. Gherghescu)
Macroscopic-microscopic method

Accounting for quantum single-particle structure and classical collective properties.

- Liquid Drop Model: $E_{LD}$
- Single-particle shell model (SPSM): energy levels vs. deformation. *Two-center shell model for fission and fusion.*
- Shell correction method: $\delta E = \delta U + \delta P$
- Total deformation energy: $E_{def} = E_{LD} + \delta E$

The potential of SPSM Hamiltonian should admit the drop eq. $\rho = \rho(z)$ as an equipotential surface. *Semi-spheroidal shape*, allows to obtain analytical results for atomic clusters on a surface.
Historical milestones (I)

- 1878 John William Strutt (Lord Rayleigh): LDM
- 1935 Carl F. von Weizsäcker: Mass formula (binding energy)
- 1928 G. Gamow explained $\alpha$-decay — quantum tunnelling
- 1939 O. Hahn, Lise Meitner, F. Strassmann: induced fission. Explained with N. Bohr’s LDM
- 1940 G.N. Flerov & K.A. Petrzhak: spontaneous fission
- 1960 V. Goldansky predicted various kinds of proton rad.
- 1962 V. Karnaukhov: $\beta$-delayed proton radioactivity
Historical milestones (II)

- 1962 S.M. Polikanov: fissioning shape isomers
- 1967 V.M. Strutinsky: shell & pairing corrections
- 1969 U. Mosel & W. Greiner: two-center shell model, prediction of superheavy nuclei, initiated creation of GSI
- 1980 S. Hofmann: proton radioactivity
- 1981 C. Signarbieux: cold fission
- 1984 H.J. Rose and G.A. Jones detected cluster radioactivity
- 1989 Fine structure of $^{14}$C decay, SOLENO at IPN Orsay
The dawn of the Nuclear Age

Wilhelm Conrad Roentgen (Nobel prize 1901) discovered the X-rays in December 1895. Radioactivity (coined by Marie Curie) of an uranium salt was discovered by Antoine Henri Becquerel in March 1896. Marie Sklodowska Curie and Pierre Curie realized it is an atomic property of matter. Th is also emitter. Ra and Po are million times much stronger. Becquerel, Marie and Pierre Curie shared the Nobel prize 1903. In 1911 Marie Curie received the 2nd Nobel prize for discovery of Ra and Po.

Ernest Rutherford (1871-1937, 1908 Nobel prize) gave the names $\alpha$ and $\beta$ radioactivity. From scattering experiments (1911) he deduced that atomic particles consisted primarily of empty space surrounding a central core called nucleus. He transmuted one element into another, elucidated the concepts of the half-life and decay constant. By bombarding N with $\alpha$-particles produced oxygen. The atomic nucleus was discovered around 1911.
LDM and Quantum Tunneling

John William Strutt (Lord Rayleigh) (Nobel Prize, 1904) Book


The critical ratio length/width = 4.5

Niels Bohr (Nobel Prize, 1922)

Lord Rayleigh, Phil. Mag. 14 (1878) 184
C.F. von Weizsäcker, Z. Phys. 96 (1935) 431

B & W 1939: fission was more likely to occur with $^{235}$U than $^{238}$U.

We use LDM: W.D. Myers and W.J. Swiatecki, Nucl. Phys. A 81 (1966) 1

Tunneling, first application of quantum theory to nuclei: G. Gamow, Z. Phys. 51 (1928) 204. Explained $\alpha$-decay.
Discovery of nuclear fission (1939)

**Induced fission**: Otto Hahn (Nobel prize 1944), Lise Meitner and Fritz Strassmann — E. Fermi Award 1966. O. Frisch, Lise Meitner’s nephew, borrowed the name Fission from biology (cell division).

**Spontaneous fission** (1940): G.N. Flerov and K.A. Petrunzhak
Nuclear shape parametrization

Collective coordinates: separation distance of the fragments, neck radius, mass and charge asymmetry, deformation of each fragments, etc.

Expansion in terms of spherical harmonic, $Y_{\lambda \mu}$, or Legendre polynomial, $P_m$.

A point on the surface

$$R(\theta, \varphi) = R_0 \left[ 1 + \alpha_{00} + \sum_{\lambda=1}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \alpha^*_{\lambda \mu} Y_{\lambda \mu}(\theta, \varphi) \right]$$

$$R_0 = r_0 A^{1/3} \quad \alpha_{00} \text{ determined from volume conservation } V = (4/3) \pi R_0^3.$$ 

Radius is real: $(\alpha_{\lambda \mu})^* = (-)^\mu \alpha_{\lambda -\mu}$. $\lambda = 2$ quadrupole deformation.

$\lambda = 3$ octupole deformation. $\lambda = 4$ hexadecapole deformation.
Spheroidal deformation

Not suitable for fission or fusion

Lengths in units of $R_0 = 1.2249A^{1/3}$ fm.

Vol. conserv. $\omega^2 \omega_z = (\omega^0_0)^3 \hbar \omega^0_0 = 41A^{-1/3}$ MeV $\hbar^2/M \approx 41.5$ MeV·fm$^2$

Quadrupolar deformation: $\varepsilon = \frac{3(c-a)}{2c+a}$. Harmonic oscill. freq.

$\omega(\varepsilon) = \omega_0 \left( 1 + \frac{\varepsilon}{3} \right) ; \omega_z(\varepsilon) = \omega_0 \left( 1 - \frac{2\varepsilon}{3} \right)$. S. G. Nilsson 1955
Two intersected spheres. Volume conservation and $R_2 = \text{const.}$ One deformation parameter: separation distance $R$. Surface equation $\rho = \rho(z)$. Initial $R_i = R_0 - R_2$. Touching point $R_t = R_1 + R_2$.

Example: $^{232}\text{U} \rightarrow ^{24}\text{Ne} + ^{208}\text{Pb}$

Two center shell model (Frankfurt) potential

Sequence of shapes
Nucleus considered a uniformly charged drop. Two variants: LDM and Yukawa-plus-exponential (Y+EM).

LDM (surface + Coulomb) deformation energy

\[
E_{LDM} = E - E^0 = (E_s - E_s^0) + (E_C - E_C^0)
\]

\[
= E_s^0 (B_s - 1) + E_C^0 (B_C - 1)
\]

For spherical shapes \(E_s^0 = a_s (1 - \kappa I^2) A^{2/3}\); \(I = (N - Z)/A\);

\(E_C^0 = a_c Z^2 A^{-1/3}\). Nuclear fissility \(X = E_C^0 / (2E_s^0)\).

Parameters obtained by fit to experimental data on nuclear masses, quadrupole moments and fission barriers: \(a_s = 17.9439\) MeV, \(\kappa = 1.7826\), \(a_c = 3e^2/(5r_0)\), \(e^2 = 1.44\) MeV·fm, \(r_0 = 1.2249\) fm.

Shape dependent $B_s$ and $B_C$

$B_s$ is proportional with surface area $B_s = \frac{d^2}{2} \int_{-1}^{+1} \left[ y^2 + \frac{1}{4} \left( \frac{dy^2}{dx} \right)^2 \right]^{1/2} dx$

In cylindrical coordinates with -1, +1 intercepts on the symmetry axis $y = y(x)$ or $y_1 = y(x')$ is the surface equation. $d = (z'' - z')/2R_0$ – seminuclear length in units of $R_0$. Assume uniform charge density, $\rho_{0e} = \rho_{1e} = \rho_{2e}$. D.N. Poenaru et al., Comp. Phys. Comm. 16 (1978) 85, 19 (1980) 205. $K, K'$ – complete elliptic integrals of the 1st and 2nd kind. $D = (K - K')/k^2$.

$$B_c = \frac{5d^5}{8\pi} \int_{-1}^{+1} dx \int_{-1}^{+1} dx' F(x, x')$$

$$F(x, x') = \{yy_1[(K - 2D)/3] \cdot \left[ 2(y^2 + y_1^2) - (x - x')^2 + \frac{3}{2}(x - x') \left( \frac{dy_1^2}{dx'} - \frac{dy^2}{dx} \right) \right] +$$

$$K \left\{ y^2y_1^2/3 + \left[ y^2 - \frac{x - x'}{2} \frac{dy^2}{dx} \right] \left[ y_1^2 - \frac{x - x'}{2} \frac{dy_1^2}{dx'} \right] \right\} \rho^{-1}$$
Potential energy surfaces (PES) for $^{106}$Te (left) and $^{232}$Th (right)

Saddle point shapes for fissility parameter $X = 0.60, 0.70, 0.82$ ($^{170}$Yb, $^{204}$Pb, $^{252}$Cf nuclei) obtained by solving an integro-differential equation.

Macroscopic-microscopic method

\[ E_{\text{def}} = E_{\text{LDM}} + \delta E. \]

V.M. Strutinsky (Nucl. Phys. A 95 (1967) 420)

microscopic calculation of shell and pairing corrections,
\[ \delta E = \delta U + \delta P, \]
based on the deformed shell models.

Also extended to atomic cluster physics.

V.M. Strutinsky & S. Polikanov, APS 1978 Tom Bonner Prize “For their significant contributions to the discovery and elucidation of isomeric fission. Their work has vastly expanded our understanding of the role of the single particle states on the total energy of heavy deformed nuclei. Their discoveries have had a crucial impact on the possible stability of very heavy nuclei.”
Two center shell model (I)


The Hamiltonian, $H$, is a sum of the kinetic energy, $-(\hbar^2/2M)\Delta$, and two potential terms: along the axis perpendicular to the symmetry axis is an harmonic oscillator $V_\rho = (m\omega_\rho^2/2)\rho^2$, and along the symmetry axis has two-centers $-z_1$ and $+z_1$, hence $V_z =$

$$
\begin{aligned}
\frac{m\omega_z^2}{2} \left\{ 
(\ z - z_1)^2 \ , \ z > 0 \\
(\ z + z_1)^2 \ , \ z < 0
\right. 
\end{aligned}
$$
Two center shell model (II)

One can separate the variables in the Schrödinger equation $H\Psi = E\Psi$ as $\Psi(\rho, \varphi, z) = R(\rho)\Phi(\varphi)Z(z)$, where $\Phi = e^{im\varphi}/\sqrt{2\pi}$, 

$$R = \eta^{m|/2} e^{-\eta/2} L_{n_r}^{|m|}(\eta),$$

with $\eta = \rho^2/\alpha^2_\perp$ and the quantum numbers $m = (n_{\perp} - 2i)$ with $i = 0, 1, \ldots$ up to $(n_{\perp} - 1)/2$ for an odd $n_{\perp}$ or to $(n_{\perp} - 2)/2$ for an even $n_{\perp}$. $L_{n_r}^{|m|}(x)$ is the associated Laguerre polynomial and $\alpha_{\perp} = \sqrt{\hbar/m\omega_\rho}$ has the dimension of a length. The wave function in the dimension-less variable is given in terms of a Hermite function, with $\nu_n$ nonintegers.

$$\langle x|\nu_n \rangle = \begin{cases} 
  c_n e^{-x^2/2} H_{\nu_n} \left( \frac{z-z_1}{\alpha} \right) , & z > 0 \\
  (-1)^n c_n e^{-x^2/2} H_{\nu_n} \left( -\frac{z+z_1}{\alpha} \right) , & z < 0 
\end{cases}$$
Shell corrections

The total energy of the uniform level distribution

\[ \tilde{u} = \frac{\tilde{U}}{\hbar \omega_0} = 2 \int_{-\infty}^{\tilde{\lambda}} \tilde{g}(\epsilon) \epsilon d\epsilon \]

In units of \( \hbar \omega_0 \) the shell corrections are calculated for each deformation \( \varepsilon \)

\[ \delta u(n, \varepsilon) = \sum_{i=1}^{n} 2\epsilon_i(\varepsilon) - \tilde{u}(n, \varepsilon) \]

\( n = N_p/2 \) particles. Then \( \delta u = \delta u_p + \delta u_n \).
Pairing corrections

The gap $\Delta$ and Fermi energy $\lambda$ are solutions of the BCS eqs:

$$0 = \sum_{k_i}^{k_f} \frac{\epsilon_k - \lambda}{\sqrt{(\epsilon_k - \lambda)^2 + \Delta^2}}; \quad \frac{2}{G} = \sum_{k_i}^{k_f} \frac{1}{\sqrt{(\epsilon_k - \lambda)^2 + \Delta^2}}$$

$$k_i = \frac{Z}{2} - n + 1, \quad k_f = \frac{Z}{2} + n', \quad \frac{2}{G} \simeq 2\tilde{g}(\tilde{\lambda}) \ln \left(\frac{2\Omega}{\Delta}\right).$$

The pairing correction $\delta p = p - \tilde{p}$, represents the difference between the pairing correlation energies for the discrete level distribution

$$p = \sum_{k=k_i}^{k_f} 2v_k^2 \epsilon_k - 2 \sum_{k=k_i}^{Z/2} \epsilon_k - \frac{\Delta^2}{G}$$

and for the continuous level distribution $\tilde{p} = -(\tilde{g}\Delta^2)/2 = -(\tilde{g}_s\Delta^2)/4$. Compared to shell correction, the pairing correction is out of phase and smaller. One has again

$$\delta p = \delta p_p + \delta p_n,$$

and $\delta e = \delta u + \delta p$. 
Results for $^{236}\text{Pu}$ and $^{304}\text{120}$

Shell corrections for $^{236}\text{Pu}$. Two-center shell model. Remark the smoothing due to the pairing correction.

PES vs $R$ and $\eta$ for a superheavy nucleus with $Z = 120$ and $A = 304$. The valleys due to the doubly magic fragments $^{208}\text{Pb}$ and $^{132}\text{Sn}$ are shown. Such cold valleys were used in the sixtieth by Walter Greiner to motivate the search for SHs, and the development of Heavy Ion Physics worldwide and in Germany, where GSI was built. Itkis et al. exp. confirmed the supersymmetric shoulder of fission fragment mass distributions.
Shell effects explain the mass asymmetry. Nuclear shape obtained as a solution of integro-differential equation.

$^{242}\text{Cm}$ $E_{Y+EM}, \delta E_{\text{shell+pair}}, E_{\text{def}}$ PES

separation distance

$\xi = (R - R_i)/(R_t - R_i)$

mass asymmetry

$\eta = (A_1 - A_2)/(A_1 + A_2)$
242\(^{\text{Cm}}\) barrier, touching, contour

\[ (R - R_i)/(R_t - R_i) \]

\[ E_{\text{def}} (\text{MeV}) \]

\[ 242\text{Cm} \rightarrow \text{Si } + \text{Pb} \]

\[ \eta = 0 \]

\[ E_{\text{def}} (\text{MeV}) \]

\[ \delta E_{\text{shell} + \text{pairing}} \text{ contour plot} \]

in the plane \((R - R_i)/(R_t - R_i), \eta\)
$^{222}\text{Ra}$ \textbf{$E_{Y+EM}$, $\delta E_{\text{shell+pair}}$, $E_{\text{def}}$ PES}

\begin{align*}
\delta E_{\text{sh+p}} \text{ (MeV)} & \quad \eta = \frac{A_1 - A_2}{A_1 + A_2} \\
E_{\text{Y+E}} \text{ (MeV)} & \quad \xi = \frac{R - R_i}{R_t - R_i}
\end{align*}

Prediction of heavy ion radioactivity

The New Encyclopaedia Britannica: “Heavy-ion radioactivity. In 1980 A. Sandulescu, D.N. Poenaru, and W. Greiner described calculations indicating the possibility of a new type of decay of heavy nuclei intermediate between alpha decay and spontaneous fission. The first observation of heavy-ion radioactivity was that of a 30-MeV carbon-14 emission from radium-223 by H.J. Rose and G.A. Jones in 1984.”

Our models

- Fragmentation and the asymmetric two center shell model
- Alpha-decay like theory
- Numerical superasymmetric fission (NuSAF) model
- Analytical superasymmetric fission (ASAF)

D. N. Poenaru, W. Greiner (Eds):

  - *Nuclear Decay Modes*, (IOP, Bristol, 1996).
Basic relationships

Parent → emitted ion + daughter nucleus, $^A_Z \rightarrow ^{A_e}_{Z_e} + ^{A_d}_{Z_d}$

Measurable quantities

- Kinetic energy of the emitted cluster $E_k = QA_1/A$ or the released energy $Q = M - (M_e + M_d) > 0$.
- Decay constant $\lambda = \ln 2/T$ or Half-life ($T < 10^{32}$ s) or branching ratio $b_\alpha = T_\alpha/T$ ($b_\alpha > 10^{-17}$)

Model dependent quantities ($\lambda = \nu SP_s$)

- $\nu$ frequency of assaults or $E_\nu = h\nu/2$
- $S$ preformation probability
- $P_s$ penetrability of external barrier
Shape parameters: fragment separation, \( R \), and mass asymmetry \( \eta = (A_d - A_e)/A \).

Our method to estimate preformation as penetrability of internal barrier: \( S = \exp(-K_{ov}) \). DNP, WG, Physica Scripta 44 (1991) 427.

Similarly \( P = \exp(-K_s) \) for external barrier.

Action integral calculated within Wentzel-Kramers-Brillouin (WKB) quasiclassical approximation

\[
K_{ov} = \frac{2}{\hbar} \int_{R_i}^{R_t} \sqrt{2B(R)E(R)} dR
\]

\( E \) – Potential barrier

\( B = \mu \) – Nuclear inertia = reduced mass for \( R \geq R_t \)
Analytical SuperAsymmetric Fission

Systematic search for cluster emitters: $10^5$ combinations parent - emitted cluster. WKB approximation.

$$T = \left[\frac{\hbar \ln 2}{(2E_v)}\right] e^{\text{exp}(K_{ov} + K_s)}$$

$$K_{ov} = 0.2196\left(\frac{E^0_b A_e A_d}{A}\right)^{1/2} (R_t - R_i) \left[\sqrt{1 - b^2} - b^2 \ln \frac{1 + \sqrt{1 - b^2}}{b}\right]$$

$$K_s = 0.4392\left[(Q + E_v)A_e A_d / A\right]^{1/2} R_b J_{rc}; \quad b^2 = E_v / E^0_b$$

$$J_{rc} = (c) \arccos \sqrt{\frac{1 - c + r}{2 - c} - \left[(1 - r)(1 - c + r)\right]^{1/2}}$$

$$+ \sqrt{1 - c} \ln \left[\frac{2\sqrt{(1 - c)(1 - r)(1 - c + r) + 2 - 2c + cr}}{r(2 - c)}\right]$$

$r = R_t / R_b; \quad c = rE_c / (Q + E_v); \quad E_v = a_i(A_e)Q; \quad R_i = R_0 - R_e, R_t = R_e + R_d$

$i = 1, 2, 3, 4$ for even-even, odd-even, even-odd, and odd-odd parent nuclei.

$$R_b = R_t E_c \left\{1/2 + \left[1/4 + (Q + E_v)E_l / E^2_c\right]^{1/2}\right\} / (Q + E_v)$$

$$E^0_b = E_i - Q; \quad E_i = E_c + E_l = e^2 Z_e Z_d / R_t + \hbar^2 l(l + 1)/(2\mu R^2_t)$$
Experimental masses

Examples of time spectra

222 Ra

223 Ra

232 U
Cluster emitters

Most probable emitted clusters with different colors.

Unified approach: CF; HPR, and $\alpha$-d

Three valleys: cold-fission (almost symmetrical); $^{16}\text{O}$ radioactivity, and $\alpha$-decay

$^{234}\text{U}$ half-lives spectrum (short T up)
Experimental confirmations

Rare events in a strong background of $\alpha$ particles

Detectors:
- Semiconductor telescope + electronics
- Magnetic spectrometers (SOLENO, Enge split-pole)
- Solid state nuclear track det. (SSNTD). Cheap and handy. Need to be chemically etched then follows microscope scanning

Experiments performed in Universities and Research Institutes from: Oxford; Moscow; Orsay; Berkeley; Dubna; Argonne; Livermore; Geneva; Milano; Vienna, and Beijing.
Natural radioactive family

Compare $\alpha$ and $\beta^-$ to $^{14}$C and $^{24}$Ne decays
Systematics $T_{1/2}$: $^{14}\text{C}$, $^{18,20}\text{O}$, $^{23}\text{F}$ rad.

Calculated lines within ASAF model and exp. points

new confirm — A. Guglielmetti et al., J Phys: Conf Ser 111 (2008) 012050

Systematics $T_{1/2}$: $^{22,24,25,26}\text{Ne rad.}$

![Graph showing $T_{1/2}$ values for different isotopes of Ne, with markers for new candidates and lower limits.](attachment:image.png)

Only lower limits for $^{18}\text{O}$ and $^{26}\text{Ne}$
Systematics $T_{1/2}$: $^{28,30}\text{Mg}$, $^{32,34}\text{Si}$ rad.

Minima at $N_d = 126$

Strong shell effect

Even-odd staggering
**Strong shell effects**

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<th>$Z_e$</th>
<th>$N_e$</th>
<th>Parent - Daughter</th>
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<td>$^{32}$Si</td>
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<td>$^{238}$Pu 80</td>
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Candidates for future experiments

- $^{220,222,223}$Fr, $^{224}$Ac, and $^{225}$Th as $^{14}$C emitters
- ($^{223}$Ac emitter already measured)
- $^{229}$Th for $^{20}$O radioactivity
- $^{229}$Pa for $^{22}$Ne decay mode
- $^{230,232}$Pa, $^{231}$U, and $^{233}$Np for $^{24}$Ne radioactivity
- $^{234}$Pu for $^{26}$Mg decay mode
- $^{234,235}$Np and $^{235,237}$Pu as $^{28}$Mg emitters
- $^{238,239}$Am and $^{239-241}$Cm for $^{32}$Si radioactivity
- $^{33}$Si decay of $^{241}$Cm

**Universal curves (I)**

Approximations: \( \log S = \left[ \frac{(A_e - 1)}{3} \right] \log S_\alpha, \)
\( \nu(A_e, Z_e, A_d, Z_d) = \text{constant}. \) From fit to \( \alpha \) decay:
\( S_\alpha = 0.0160694 \) and \( \nu = 10^{22.01} \text{ s}^{-1}. \)

\[
\log T = - \log P - 22.169 + 0.598(A_e - 1)
\]

\[ - \log P = c_{AZ} \left[ \arccos \sqrt{r} - \sqrt{r(1 - r)} \right] \]
\[ c_{AZ} = 0.22873(\mu_A Z_d Z_e R_b)^{1/2}, \quad r = \frac{R_t}{R_b}, \quad R_t = 1.2249 \left( A_d^{1/3} + A_e^{1/3} \right), \quad R_b = 1.43998 Z_d Z_e / Q, \quad \text{and} \quad \mu_A = A_d A_e / A. \]

Universal curves (II)

Geiger-Nuttal plot \( T_\alpha = f(\text{range of } \alpha \text{ in air}) \)

\[ \log T = f\left(\frac{1}{Q^{-1/2}}\right) \]
Fine structure of $^{14}$C radioactivity


$\alpha$-decay, ASAF, semiemp & univ

\[
\begin{array}{cccc}
\text{Group} & \sigma-\text{ASAF} & \sigma-\text{univ} & \sigma-\text{semiemp} \\
47\ e-e & 0.402 & 0.267 & 0.164 \\
45\ e-o & 0.615 & 0.554 & 0.507 \\
25\ o-e & 0.761 & 0.543 & 0.485 \\
25\ o-o & 0.795 & 0.456 & 0.451 \\
\end{array}
\]

Poenaru, D.N., Plonski, I.H., Gherghescu, R.A., Greiner, W.,
$Z = 92 - 118$, ASAF, semiemp & univ

Vertical bars: $N_d = 126, 152, 162$

Multicluster fission (I)

True-ternary and 2 particle-accompanied fission (quaternary)

Good chance to be detected: $2\alpha$-, $3\alpha$-, and $4\alpha$-accompanied fission. $Q$-value and pot. barrier of $2\alpha$-accompanied fission is similar to $^{8}\text{Be}$-accompanied fission.


Fragmentation occurs beyond the 4.5 critical value.

Dendritic (fractal) shape by deposition of silver clusters on graphite.
Semi-spheroidal atomic cluster (I)

\[ a^2 c = 1 \] — volume conservation

\[ a = \left[\frac{(2 - \delta)}{(2 + \delta)}\right]^{1/3} \]

New shell model with striking properties of symmetry.
Maximum degeneracy at \( \delta = 2/3 \)

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Semi-spheroidal atomic cluster (II)

Figures, TOP: LDM (surface + curvature) energy of Na\textsubscript{56} semi-spheroidal cluster compared to the spheroidal one. BOTTOM: Na\textsubscript{148} cluster, pairing corrections, total deformation energy (LDM + shell and pairing corrections). Within LDM the most stable shape is a superdeformed prolate semi-spheroid ($\delta \approx 0.66$).

Summary

- The ASAF model predictions have been confirmed
- The magicity of the daughter $^{208}$Pb was not fully exploited
- New experimental searches can be performed
- The universal curves provide good estimation of half-lives
- $\alpha$-decay of superheavies are well reproduced by ASAF, UNIV and semFIS
- For atomic cluster on a surface
  - The maximum degeneracy of the new shell model occurs at a superdeformed prolate semi-spheroidal shape
  - Within LDM the most stable shape is a superdeformed prolate semi-spheroid